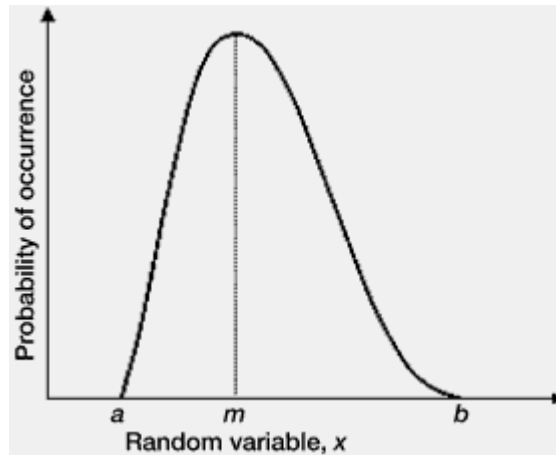


# **Probability Distribution Estimated From the Minimum, Maximum, and Most Likely Values: Applied to Turbine Inlet Temperature Uncertainty**

Modern engineering design practices are tending more toward the treatment of design parameters as random variables as opposed to fixed, or deterministic, values. The probabilistic design approach attempts to account for the uncertainty in design parameters by representing them as a distribution of values rather than as a single value. The motivations for this effort include preventing excessive overdesign as well as assessing and assuring reliability, both of which are important for aerospace applications.

However, the determination of the probability distribution is a fundamental problem in reliability analysis. A random variable is often defined by the parameters of the theoretical distribution function that gives the best fit to experimental data. In many cases the distribution must be assumed from very limited information or data. Often the types of information that are available or reasonably estimated are the minimum, maximum, and most likely values of the design parameter. For these situations the beta distribution model is very convenient because the parameters that define the distribution can be easily determined from these three pieces of information. Widely used in the field of operations research, the beta model is very flexible and is also useful for estimating the mean and standard deviation of a random variable given only the aforementioned three values. However, an assumption is required to determine the four parameters of the beta distribution from only these three pieces of information (some of the more common distributions, like the normal, lognormal, gamma, and Weibull distributions, have two or three parameters). The conventional method assumes that the standard deviation is a certain fraction of the range. The beta parameters are then determined by solving a set of equations simultaneously. A new method developed in-house at the NASA Glenn Research Center assumes a value for one of the beta shape parameters based on an analogy with the normal distribution (ref.1). This new approach allows for a very simple and direct algebraic solution without restricting the standard deviation. The beta parameters obtained by the new method are comparable to the conventional method (and identical when the distribution is symmetrical). However, the proposed method generally produces a less peaked distribution with a slightly larger standard deviation (up to 7 percent) than the conventional method in cases where the distribution is asymmetric or skewed. The beta distribution model has now been implemented into the Fast Probability Integration (FPI) module used in the NESSUS computer code for probabilistic analyses of structures (ref. 2).



*Example of a beta probability distribution with minimum value,  $a$ ; maximum value,  $b$ ; and most likely value,  $m$ . A new approach was developed for determining the beta distribution given  $a$ ,  $b$ , and  $m$ .*

This work was motivated by the problem of modeling the uncertainty in turbine inlet temperature for aeroengines given the minimum, maximum, and most likely temperature values (ref. 3). The method, therefore, offers enhanced probabilistic assessment of aeropropulsion components and systems subject to nondeterministic thermal loads. However, the method is applicable to the general problem of estimating the parameters of the beta distribution for any random variable where the minimum, maximum, and most likely values are available. Furthermore, this work demonstrated the utility of the beta distribution for these types of problems in probabilistic engineering analyses.

## **References**

1. Holland, Frederic A., Jr.: A Simple Method for Estimating the Parameters of the Beta Distribution Applied to Modeling Uncertainty in Gas Turbine Inlet Temperature. ASME Paper GT-2002-30295, 2002, pp. 627-633.
2. Tong, Michael T.: A Probabilistic Assessment of Advanced Aeropropulsion Technologies. ASME Paper GT-2004-53485, 2004.
3. Tong, Michael T.: A Probabilistic Approach to Aeropropulsion System Assessment. NASA/TM--2000-210334, 2000. <http://gltrs.grc.nasa.gov/cgi-bin/GLTRS/browse.pl?2000/TM-2000-210334.html>

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